OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI) 4768

Statistics 3

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
None

Wednesday 17 June 2009
Morning

Duration: 1 hour 30 minutes

4768

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

Turn over

1 Andy, a carpenter, constructs wooden shelf units for storing CDs. The wood used for the shelves has a thickness which is Normally distributed with mean 14 mm and standard deviation 0.55 mm. Andy works to a design which allows a gap of 145 mm between the shelves, but past experience has shown that the gap is Normally distributed with mean 144 mm and standard deviation 0.9 mm. Dimensions of shelves and gaps are assumed to be independent of each other.

(i) Find the probability that a randomly chosen gap is less than 145 mm. **[3]**

(ii) Find the probability that the combined height of a gap and a shelf is more than 160 mm. **[3]**

A complete unit has 7 shelves and 6 gaps.

(iii) Find the probability that the overall height of a unit lies between 960 mm and 965 mm. Hence find the probability that at least 3 out of 4 randomly chosen units are between 960 mm and 965 mm high. **[7]**

(iv) I buy two randomly chosen CD units made by Andy. The probability that the difference in their heights is less than h mm is 0.95. Find h. **[5]**

2 Pat makes and sells fruit cakes at a local market. On her stall a sign states that the average weight of the cakes is 1 kg. A trading standards officer carries out a routine check of a random sample of 8 of Pat's cakes to ensure that they are not underweight, on average. The weights, in kg, that he records are as follows.

0.957 1.055 0.983 0.917 1.015 0.865 1.013 0.854

(i) On behalf of the trading standards officer, carry out a suitable test at a 5% level of significance, stating your hypotheses clearly. Assume that the weights of Pat's fruit cakes are Normally distributed. **[9]**

(ii) Find a 95% confidence interval for the true mean weight of Pat's fruit cakes. **[4]**

Pat's husband, Tony, is the owner of a factory which makes and supplies fruit cakes to a large supermarket chain. A large random sample of n of these cakes has mean weight $\bar{x}$ kg and variance 0.006 kg^2.

(iii) Write down, in terms of n and $\bar{x}$, a 95% confidence interval for the true mean weight of cakes produced in Tony's factory. **[3]**

(iv) What is the size of the smallest sample that should be taken if the width of the confidence interval in part **(iii)** is to be 0.025 kg at most? **[3]**

3 A company which employs 600 staff wishes to improve its image by introducing new uniforms for the staff to wear. The human resources manager would like to obtain the views of the staff. She decides to do this by means of a systematic sample of 10% of the staff.

(i) How should she go about obtaining such a sample, ensuring that all members of staff are equally likely to be selected? Explain whether this constitutes a simple random sample. **[5]**

At a later stage in the process, the choice of uniform has been reduced to two possibilities. Twelve members of staff are selected to take part in deciding which of the two uniforms to adopt. Each of the twelve assesses each uniform for comfort, appearance and practicality, giving it a total score out of 10. The scores are as follows.

Staff member	1	2	3	4	5	6	7	8	9	10	11	12
Uniform A	4.2	2.6	10.0	9.0	8.2	2.8	5.0	7.4	2.8	6.8	10.0	9.8
Uniform B	5.0	5.2	1.4	2.8	2.2	6.4	7.4	7.8	6.8	1.2	3.4	7.6

A Wilcoxon signed rank test is to be used to decide whether there is any evidence of a preference for one of the uniforms.

(ii) Explain why this test is appropriate in these circumstances and state the hypotheses that should be used. **[4]**

(iii) Carry out the test at the 5% significance level. **[8]**

4 A random variable X has probability density function $\mathrm{f}(x) = \dfrac{2x}{\lambda^2}$ for $0 < x < \lambda$, where λ is a positive constant.

(i) Show that, for any value of λ, $\mathrm{f}(x)$ is a valid probability density function. **[3]**

(ii) Find μ, the mean value of X, in terms of λ and show that $\mathrm{P}(X < \mu)$ does not depend on λ. **[4]**

(iii) Given that $\mathrm{E}(X^2) = \dfrac{\lambda^2}{2}$, find σ^2, the variance of X, in terms of λ. **[2]**

The random variable X is used to model the depth of the space left by the filling machine at the top of a jar of jam. The model gives the following probabilities for X (whatever the value of λ).

$0 < X \leqslant \mu - \sigma$	$\mu - \sigma < X \leqslant \mu$	$\mu < X \leqslant \mu + \sigma$	$\mu + \sigma < X < \lambda$
0.18573	0.25871	0.36983	0.18573

A sample of 50 random observations of X, classified in the same way, is summarised by the following frequencies.

4	11	20	15

(iv) Carry out a suitable test at the 5% level of significance to assess the goodness of fit of X to these data. Explain briefly how your conclusion may be affected by the choice of significance level. **[9]**